SPACE DIVISION MULTIPLE ACCESS (SDMA) IN AN IEEE 802.11n WLAN

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ABSTRACT

The impetus of the present study is to describe a novel approach compatible with next-generation Wireless Local Area Networks (WLANs) IEEE 802.11n. By applying proposed MIMO (Multiple Input – Multiple Output) transmission for 802.11n WLAN, which uses multiple transmitting and receiving antennas as well as Space Division Multiple Access (SDMA) technology, high data rate of more than 240 Mbits/s on air is achieved, for 20 MHz channel bandwidth. Our proposed scheme consists of simultaneously designing downlink beamformers to multiple co-channel groups by minimizing total transmit power under the constraint on providing at least a prescribe received Signal-to-Interference – plus – Noise Ratio (SINR) to each intended receiver. We assume that the instantaneous channel gains for all users are known at the Access Point (AP). Our simulation results show substantial gain for our proposed strategies in IEEE 802.11n wireless access networks.

1. INTRODUCTION

The future growth of wireless communications in the decade to come is focus on a large variety of multimedia applications. Requirements of high spectral efficiency, high throughput and low latency are essential for the wireless delivery of such content in real time applications. Due to their low cost and high data transmission rate, IEEE 802.11 based WLANs have become increasingly popular and especially WiFi (Wireless Fidelity). Currently 802.11 Task Group n (802.11 TGn) makes an effort to define modifications to physical and MAC (Medium Access Control) layer aiming at providing a minimum of 100 Mbits/s throughput at the MAC SAP (Service Access Point) which means that the transfer rate over air will be 200 Mbps/s [1]. Aiming at keeping the cost down and at the same time easing backward compatibility, IEEE 802.11n proposes the reusage of legacy technologies such as the Orthogonal Frequency Division Multiplexing (OFDM) and Quadrature Amplitude Modulation (M-QAM). In order to increase the transfer rate of wireless system, MIMO technology and wider bandwidth channels are required [1,2,3].

SDMA is a communication technique that maximizes the number of users which communicate with the same AP simultaneously in the same frequency band with very little interference on each other. By exploiting SDMA and multiuser diversity, beamforming provides high spectral efficiency in situations involving a higher number of users. Optimal spatial mode depends not only on the physical channel properties but also on system load and deployment. In [4,5], MIMO-SDMA for a multi-user downlink narrowband system is consider taking multi-user interference into account. The transmitters and receivers exploit the CSI (Channel State Information) to form beamforming weights for transmission and reception in order to minimize total Tx power satisfying the SINR requirements at all users. The proposed solution is an iterative link by link algorithm and therefore it has a large computation cost. The performance in [5] is evaluated taking several APs. Each AP serves one or two users. Various references for downlink multi-user MIMO-OFDM systems have been proposed. Qualcomm's 802.11n proposal includes a physical layer (PHY) strategy, mentioned as Eigenvector Steering (ES), which calculates the optimum transmit and receive parallel stream vectors for each sub-channel from Singular Value Decomposition (SVD) of channel matrix [6]. The same philosophy is documented in [7] where the transmission scheme is named Eigenbeam – Space Division Multiplexing (E_SDM).

The core idea of our proposed design is the following: consider an AP a, where the transmitter is equipped with M_t antennas and there are K users. In order to increase throughput, SDMA is used. According to their locations, the users are divided in g groups. SDMA technique accommodates the g groups using the same frequency within the same cell simultaneously by constructing a spatially independent channel to each group and transmitting signals in parallel. Consequently, frequency utilization can greatly improve. Afterwards, downlink beamformers are designed for the co-channel groups under quality of services (minimum attained signal to interference plus noise ratio at each receiver). The goal is to minimize total transmission power and thus leakage to neighboring co-channel groups. We define the operation frequency of AP a f_a at the center of channel band. This is not a link-by-link optimization problem and therefore hasn't large computation cost. For each group, multi-user diversity can be exploited to find a subset of good sub-carriers to meet Quality of Services (QoS) requirement. Following the optimal beamformers at AP, the different sub-carriers of each group can be allocated to different users (OFDMA). By adaptively employing different modulation modes on the sub-carriers according to SINR, we enhance the system performance. The drawback in such group optimization design is that the receivers are found at different locations, near or away from AP. The constraint of minimum guarantee SINR for each receiver must be satisfied. In order to improve the performance of our system, multiple antennas at the receiver are proposed. Coherent combination of diversity paths increases SNR in comparison to just a single antenna receiver. This growth of SNR is called "array gain".

The AP and users exploit CSI to form beamforming weights. In sort-rang scenario like WLAN, CSI is feasible because of low mobility. The general power minimization problem (subject to SINR constraints) of simultaneously designing beamformers for several co-channel multicast groups and frequency flat channel was studied in [8]. This problem can be formulated as a convex optimization problem. However, the optimum solution is considered for narrowband and single antenna receivers. We extended this design into a wideband with multiple antenna receivers system. The efficiency of the proposed solution is studied by simulation with real values (non-normalized) derived from IEEE 802.11n channel model [9]. The simulation includes all the main propagation effects such path loss, shadowing, delay spread, spatial correlation and Doppler effects.

The paper is organized as follows: in the next section, we define the system model. In section III downlink beamforming MIMO-SDMA algorithms are developed. In section IV performances of proposed algorithm are extensively simulated and are finally compared to currently used strategies. Section V concludes this article.

2. SYSTEM MODEL

TGn supports 20 MHz bandwidth where the spectrum is limited and 40 MHz with two adjacent spectral channels. This technique which combines two adjacent channels of 20 MHz into one of 40 MHz is called channel bonding. The transmission of 200Mbits/s in 20 MHz yields a bandwidth efficiency of 10 b/s/Hz. The high data rate digital signal is converted into 48 overlapping low data rate (sub-channels) using an Inverse FFT. Each sub-channel occupies 312.5 KHz and is not individually filtered. The total number of generated sub-carries is 64. Physical layer (PHY) modes in 20 MHz channel width with different coding and modulation schemes are present in Table I. Minimum receiver sensitivity is the power at the antenna port of receiver for which packet error rate (PER) is less than 1% for PHY layer service data unit(PSDU) or for payload length equal to 4095 bytes.

TABLE I. MODE –DEPENDENT PARAMETERS

Mode	Modulation	Coding Rate (R)	Coded bits per sub-carrier	Minimum Sensitivity (dBm)
1	BPSK	1/2	1	-80
2	QPSK	1/2	2	-77
3	QPSK	3/4	2	-75
4	16-QAM	1/2	4	-72
5	16-QAM	3/4	4	-68
6	64-QAM	2/3	6	-64
7	64-QAM	3/4	6	-63
8	64-QAM	5/6	6	-62

We study a system with a single AP a and K users. Both AP and each user are equipped with array antennas M_t and M_r respectively. Consider a total of $1 \leq G \leq K$ groups $\{G_1\ G_2\ \dots\ G_G\}$, where G_g contains the indices of receivers participating in group g, $g \in \{1\ 2\ \dots\ G\}$. Each receiver belongs to a single group $G_g \cap G_m = \{\ \}$ with $g \neq m$ and $\sum_{g=1}^{G}|G_g| = K$. The schematic model for the AP is depicted in figure 1. At the transmitter, all user' packets are sent to group construction module. This module drives the data into convenient group and resource allocation module find the number of assigned sub-carriers at each user from the total number of data sub-carriers. The binary data are encoded by Forward Error Correction coding (FEC) and interleaving (π) is added to be the transmitted information robust against burst errors. Sub-carrier allocation algorithm dynamically assigns the sub-carriers of each group to different users. In mapper, the binary data is either adaptively divided into groups of 1,2,4,6 bits and converted into complex number representing BPSK,QPSK, and 16 , 64-QAM constellation. OFDM system is implemented in discrete time using an Inverse FFT (IFFT) to acts as modulator. In the guard time, the OFDM symbol is cyclically extended to avoid inter-carrier interference. The guard interval with cyclic extension is called Cyclic Prefix (CP). A Digital to Analog Converter (DAC) with a Low Pass Filter (LPF) transforms the digital data to analog. RF (Radio Frequency) modulation is performed and the signal is up-converted to transmission frequency f_a, at the center of band. RF signals of each group are multiplied by complex weights and summed to feed the M_t antenna elements (beamforming). In figure 1, $G \leq M_t$ beamforming modules are used and therefore up to G beams are possible to be constructed. At the receiver, the uncoupled operation is evaluated to decode the information bits for every user belong to group g. To provide high data rate, Automatic Modulation and Coding (AMC) are adopted on every sub-channel. According to SNR_n^g for the n sub-carrier belongs to group g, different modulation schemes of AMC can be implemented. If SNR_n^g threshold is guaranteed, no packet errors are assumed. The packet loss only happens when the buffer of user is overflowed. The most critical assumption is the availability of CSI at the transmitter and at the receiver. Accurate CSI may be easy to obtain when the channel changing slowly as is mentioned above. It's much more difficult in situations where the channel is changing rapidly as in cellular telephone applications. In PHY, synchronization, pilot and signal segment are added to MAC Packet Data Unit (MPDU). Each user estimates the propagation channel from pilot segments. The CSI is known at AP by using an up-link to return channel measurement collected during the transmission of downlink training sequences at 48 data sub-carriers. In IEEE 802.11n explicit feedback is proposed in the TDD mode,ie each user sends to AP the MIMO channel coefficients. The feedback of CSI Matrices can be immediate or aggregate. The structure of CSI Matrices Report includes all sub-carriers.

3. DOWNLINK BEAMFORMER DESIGN

The receiver antenna arrays have a properly determined beam pattern. They steer the beams to enhance the total power in all reflected paths at a scattering environment. Array gain is achieved via coherent combining of the

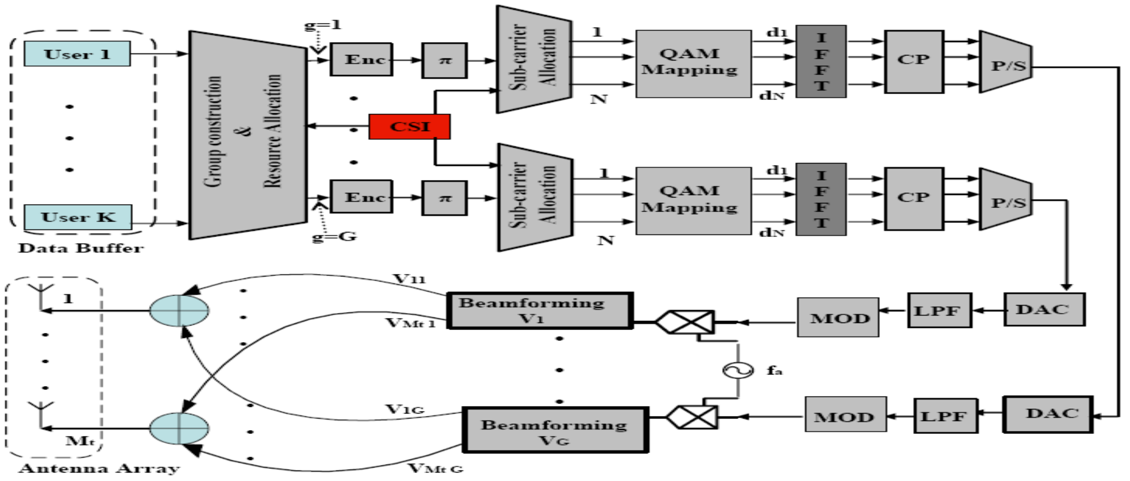

Fig. 1 The PHY transmitter model for AP

signal paths. The proposed strategy is based on ES or E-SDM. H_k^n can be diagonalized as $H_k^n = U_k^n D_k^n (V_k^n)^H$. The SVD is an appropriate way of diagonalizing the matrix H_k^n which leads to a number of parallel channels (eigen modes). The columns $u_{ki}^n(v_{ki}^n)$ of $U_k^n(V_k^n)$ are the optimum weights of Rx antenna arrays (Tx antenna arrays) for ith eigen mode [10]. E-SDM forms beams using eigen-vectors and can configure a spatially orthogonal MIMO channel that is a channel without crosstalk. Therefore, receiver antenna arrays form beams using eigen-vectors and expect to capture all possible orthogonal spatial streams derived from scatters which are found to the neighborhood of transmitter and receiver. SVD must be evaluate at the receiver. The calculation load increases proportionally according to the number of sub-carriers. But the channel in frequency domain is smooth. The adjacent sub-carriers are highly correlated because we multiply all sub-carriers by the same complex weights. This correlation reduces the calculation load. One can estimates U_k^n matrix in one specific sub-carrier n by interpolation and smoothing over adjacent sub-carriers. In E-SMD technique, the beamformer applied at the transmitter is the matrix V_k^n at each sub-carrier. The equivalent channel after beamforming is not still remaining smooth leading to higher computation load and high power consumption. The operation frequency of our downlink beamforming optimization problem is defined at the center of band, where according IEEE 802.11n subcarrier is null for measuring the noise level at the receiver. Multipath delays cause frequency selective fading and par consequence the estimated channel matrices H_k^1 to H_k^N perhaps present large gain variation in the same band. To overcome these problems, the transmitted and the received weight vectors are obtained from average channel matrix $H_k = \frac{1}{N}\sum_{n=1}^{N} H_k^n$. If Rx matrix is U_k $M_r \mathrm{x} M_r$, v_g $M_t\mathrm{x}1$ is beamforming weight vector applied to the M_t antenna elements and the noise at receiver k is zero mean with variance σ_k^2 then the posed problem is to generate an optimal downlink beamforming at AP a, minimizing at the same time the total transmitted power and guarantying prescribed SINR constraints γ_k at each user of group g.

$$\mathcal{Q} \quad \min_{\{v_g \in C^{M_t}\}_{g=1}^{G}} \sum_{g=1}^{G} \|v_g\|_2^2$$

$$\text{s.t.} \quad \frac{|U_k^H\, H_k\, v_g|^2}{\sum_{l \neq g} |U_k^H\, H_k\, v_l|^2 + \sigma_k^2\, \|U_k\|^2} \geq \gamma_k$$

$$\|U_k\|^2 = 1 \; \forall k \in \{1 \ldots K\} \; \forall g \in \{1 \ldots G\}$$

This problem was found NP-hard for general channel vector [8, 11]. Let's introduce $r_k = U_k^H\, H_k$ $M_r \mathrm{x} M_r$ complex vector, define $V_g = v_g\, v_g^H$, $R_k = r_k^H\, r_k$ and use $|r_k\, v_g|^2 = v_g^H\, r_k^H\, r_k\, v_g = tr\,(V_g\, R_k)$. The problem $\mathcal{Q}$ is transformed as

$$\mathcal{Q}' \quad \min_{\{V_g \in C^{M_t \mathrm{x} M_t}\}_{g=1}^{G}} \sum_{g=1}^{G} tr\,(V_g)$$

$$\text{s.t.} \quad \frac{tr\,(R_k\, V_g)}{\sum_{l \neq g} tr\,(R_k\, V_l) + {\sigma_k}^2} \geq \gamma_k$$

$$V_g \geq 0 \quad V_g = V_g^H \quad rank\,(V_g) = 1$$

$$\forall k \in \{1 \ldots K\} \; \forall g \in \{1 \ldots G\}$$

The constraint $rank\,(V_g) = 1$ is applied from the fact that $V_g = v_g\, v_g^H$. Constrains $V_g \geq 0$ and $V_g = V_g^H$ mean that V_g is symmetric, positive, semidefinite matrix. In general case, the constraint $\{rank\,(V_g) = 1\}_{g=1}^{G}$ is not convex [11]. As shown by Bengtsson and Ottersten, the above relaxation is guaranteed to have at least one optimal solution which is rank one [12]. Ignore the associated non convex constraints, the original non-convex Quadratically Constrained Quadratic Programming (QCQP) problem $\mathcal{Q}$ relaxed to a suitable Semi Definite Programming problem (SDP). We introduce K real non-negative "slack" variables $\{s_k\}_{k=1}^{K}$ and we underline the fact that the terms in denominator of linear inequalities are all non-negative; we take the relaxation problem $\mathcal{R}$.

$$\mathcal{R} \quad \min_{\{V_g \in C^{M_t \mathrm{x} M_t}\}_{g=1}^{G}} \sum_{g=1}^{G} tr\,(V_g)$$

$$\text{s.t.}\; tr\,(R_k V_g) - \gamma_k \sum_{l \neq g} tr\,(R_k V_l) - s_k = \gamma_k {\sigma_k}^2$$

$$V_g \geq 0 \quad V_g = V_g^H \quad s_k \geq 0$$

$$\forall k \in \{1 \ldots K\} \; \forall g \in \{1 \ldots G\}$$

The relaxation technique can be interpreted as the Lagrangian dual of the dual of the original problem because it gives a lower bound for the original problem. The first advantage of using SDP is that problem $\mathcal{R}$ is a convex optimization problem and hence it has not local minima. The second one is that problem can be efficiently solved by any SDP solver, such as SeDuMi [13], based on interior point methods. Problem $\mathcal{R}$ can be expressed in the standard prima form used in SeDuMi. It consists of G variables $M_t \mathrm{x} M_t$ and M inequality constraints. SeDuMi is an iterative algorithm. Therefore, the complexity per iteration is $O\left(\left(GM_t^2+K\right)^3\right)$ and for solution accuracy ω SeDuMi gives $O\left(\sqrt{GM_t^2+K}\,\log(\frac{1}{\omega})\right)$ worst-case iteration bound. The relaxed problem $\mathcal{R}$ provides only lower bounds on the optimal solution $\{v_g^{\mathrm{opt}}\}_{g=1}^{G}$ due to the fact that V_g^{opt} will not be rank – one in general.

In [14] randomization is proposed for computing feasible points in a QCQP problem. If x is the variable matrix of the original problem and $X = x\,x^T \geq 0$ the variable of the relaxed problem then x is selected as a Gaussian variable with $x \backsim N(x, X)$Afterwards, x will solve the QCQP "on average". A good feasible point can be obtained by trying enough x. Inspired by the above method, a randomization procedure is employed in [8] to generate candidate beamforming vectors v_g .This procedure is mentioned as $\mathrm{rand}\,\mathcal{C}$. At the beginning, SVD is used in $V_g^{\mathrm{opt}} = U\,\Sigma^{\frac{1}{2}}\,U^H$ and $v_g = U\,\Sigma^{\frac{1}{2}}\,w_g$ is put, where w_g is a Gaussian variable with $w_g \backsim N(0,1)$ to insure that $E\left[v_g\,v_g^H\right] = V_g^{\mathrm{opt}}$. However, the candidate beamforming vectors must satisfy the constraints of original problem $\mathcal{Q}$. In this way, for each candidate set of beamforming vectors, a multi-group power control $(\mathcal{MGPC})$ problem is solved.

$$\mathcal{MGPC} \qquad \min_{\{P_g \in \mathcal{R}\}_{g=1}^{G}} \sum_{g=1}^{G} \beta_g\,P_g$$

$$\text{s.t.} \qquad \frac{P_g\,\alpha_{g,k}}{\sum\limits_{l \neq g} P_l\,\alpha_{l,k} + \sigma_k^2} \geq c_k$$

$$\forall\, g \in \{1\,\ldots\,G\} \quad P_g \geq 0$$

where $\beta_g = \|v_g\|_2^2$, $\alpha_{g,k} = |v_g^H R_k v_g|$ and P_g denotes the power boost factor for multicast group g. This is a Linear Program (LP) and can be solved by SeDuMi with the computational cost being negligible.

4. SIMULATION RESULTS

For validation reasons, we apply the proposed algorithms to a cell which is represented by a circle radius R and served by an AP at the center of cell. Transmit power of AP is $P_t = 250mW$ while cell radius is considered to be equal to $R = 100m$. The system simulation is implemented in MATLAB environment. We assume that the number of transmit antennas at the AP is four $(M_t = 4)$, the number of receive antennas at user station is four $(M_r = 4)$. Uniform Linear Arrays with half wavelength spacing are used at both ends. Operation band is 20 MHz in 5GHz spectrum and the number of data sub-carriers is $N = 48$. We consider the Ricean channe model as the propagation model. If H_{LOS} is a rank-one matrix corresponding to one LOS path and H_{NLOS} represents a component of the multi-path scatters, then, the Rice factor K is the strength of $\|H_{LOS}\|_2^2$ relative to $\|H_{NLOS}\|_2^2$. Channel matrix H_{LOS} is computing with a break point $d_{BP} = 5m$, a path loss exponent $n = 3$, a shadowing deviation $\sigma_{\psi dB} = 4\,dB$ and a Ricien K-factor $K = 3$ according IEEE 802.11n channel model in B environment [9]. Channel matrix H_{NLOS} is simulated as model B with MATLAB implementation available from L. Schumacher [15]. Users are placed in random locations, following a Uniform Distribution. We run downlink beamforming optimization problem (SDP relaxation, $\mathrm{rand}\,\mathcal{C}$ randomization and power control $\mathcal{MGPC}$) for 107 samples of time varying channel B due to Doppler effect. In indoor wireless systems, transmitters and receivers are stationary and people are moving. To solve the $\mathcal{MGPC}$ problem 1000 Gaussian randomization samples are generated. For the simulated channel matrices $H_k \quad \forall k \in \{1\,\ldots\,K\}$, we use the narrowband assumption, which implies that the signal seen at the receiver is a summation of all channel taps. This assumption is valid for our system because H_k is the average channel matrix of $N = 48$ narrowband channel matrices H_k^n . The received $SINR$ constraints are set to $\gamma_k = 15\,dB$ (minimum $SINR$ for mode 1) and the noise power at user k $\sigma_k^2 = -95\,dBm$. Adaptive modulation is used selecting the highest modulation based on tables I. $SINR$ is computed from downlink beamforming optimization algorithms by averaging over all 107 channel samples. Average PHY data rate is achieved if we allocate the same number of sub-carriers to each user. In IEEE 802.11n channel model, clustered structure of the propagation environment is assumed. More specifically, there is significant local scattering around both the AP and users, causing uncorrelated fading at each end of the MIMO link. Angular characteristic of users' position

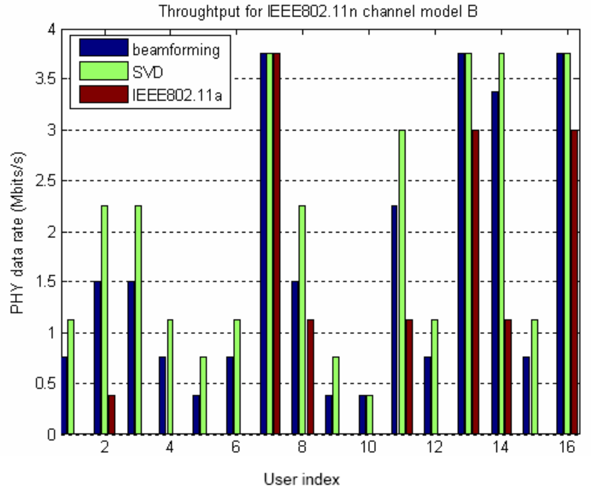

Fig. 2 Throughput for channel model B

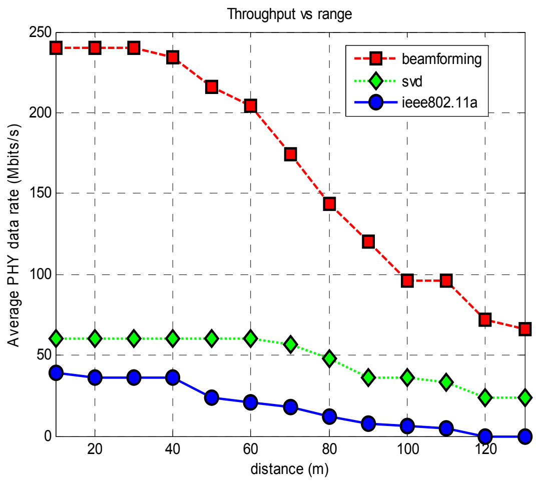

Fig. 3 Throughput versus range for LOS/NLOS channel model

doesn't offer information for simulation. Only distance between AP and users is inserted for computing H_k. Figure 3 shows the average physical data rate over 107 channel varying samples when AP broadcast to 16 users in NLOS environment (channel model B). Computer simulations are performed for the proposed method, ES or E_SDM in 4x4 MIMO and IEEE 802.11a standard. The obtained power of each sample (in our scenario, mean power is 0.2424 W) is used to compute throughput of ES and IEEE 802.11a configurations. The two first techniques achieve a noticeable improvement over the IEEE 802.11a . ES achieves a total throughput improvement of 0.2286 in comparison to our scheme in NLOS environment. However, the second technique can be implemented more easily; one weight vector is applied to antenna array, while 16 weight matrices 4x4 are required in first technique. Afterwards, we consider a wireless scenario incorporating $K = 4$ users in order to compare the three techniques in LOS/NLOS environment. Angular users' direction (θ) is 0º,180º,90º and 270º. We suppose that users are equaled by AP. Knowledge of users' direction permits gathering users in four multicast groups $(G = 4)$. Figure 4 shows the average physical layer data rate as function of distance. Improvement of beamforming technique is noticeable. Channel bonding technique permits 12 zeroed sub-carriers to be used as data sub-carriers and therefore the maximum average PHY data rate for proposed scheme is 540 Mbits/s

5. CONCLUSIONS

In this paper, we studied and developed a strategy compliant with IEEE 802.11n proposal. The estimation of channel and the combination of SDMA technique, MIMO transmission , channel bonding permit to achieve data rate as high as 540 Mbps transmission by radio, fully nine times greater than the maximum available with 802.11 a/g in LOS/NLOS environment.

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