

# Multicast Transmission over IEEE 802.11n WLAN

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*Abstract*—With the advent of low-cost WLAN devices, the delivery of multimedia content is highly desirable. Such applications require high throughput and near-real time for quality viewing. The use of next-generation WLAN 802.11n and physical layer multicast transmission for a wide range of wireless terminals introduces many significant challenges. The impetus of the present study is to describe a cross-layer approach, enabling techniques as beamforming, MIMO antennas, OFDM and low latency MAC operation. Our simulation results show a substantial improvement in network performance for our proposed strategies in IEEE802.11n wireless access networks.

## I. INTRODUCTION

Multicasting is the ability of a communication network to accept a single message from an application and to deliver copies of the message to multiple recipients at different locations. It allows transmission and routing of packets to multiple destinations using fewer network resources. Multicasting is a more efficient method of supporting group communication than unicasting or broadcasting. Wireless Local Area Network (WLAN) devices are well established due to low cost. PDAs, laptops and cell phones include a WLAN modem as standard. Service Providers have great interest in supporting services with a large number of clients attempting the same content from a single Access Point (AP). Wireless multicast support many important multimedia applications as digital video libraries, distance learning, company training, electronic commerce, on line games, redistribution of TV, etc.

Video streaming is very different from data communication due to inherent delay constraints; as late arriving data are not useful to the video decoder. In this paper, we focus on IEEE802.11n which transmit higher physical layer data rate and improve MAC efficiency. IEEE802.11n is an extension and completely downward compatible with the existing WLAN 802.11 a/g. The new standard proposes the reuse of legacy technologies such as Orthogonal Frequency Division Multiplexing (OFDM) and Quadrature Amplitude Modulation (M-QAM). Mean transfer rate over air will be 200 Mbts/sec [1]. Higher data rate is achieved mainly by using multiple antennas at the both transmitting and receiving end and hence the name MIMO (Multiple-Input, Multiple-Output). The high data rate digital signal is converted into 48 orthogonal overlapping low data rate using an Inverse Fast Fourier Transform (IFFT). Each subchannel occupies 312.5 KHz and is not individually filtered. In addition, there are four pilot channels and one null in the center of the band. The available bandwidth is 20 or 40 MHz (two adjacent channels) in 2.4 GHz ISM (Industrial, Scientific and Medical) or 5 GHz UNII

(Unlicensed National Information Infrastructure) band.

MAC layer use the simple and robust carrier sense multiple access with collision avoidance(CSMA/CA) technique for medium access which has contributed to the success of 802.11. However, this scheme imposes a considerable penalty on efficiency. Good performance is only obtained when a few users compete for access to the wireless media. Multicast overcomes this inefficiency by reducing the contention. IEEE 802.11n improves preamble overheads at data and controls frame transmission as well as interframe spacing. However, IEEE 802.11n does not provide a reliable multicast service. The mechanism of Acknowledge (ACK) is only supported for unicast mode. The data packets are not acknowledged and hence not retransmitted on the MAC layer. Therefore, application layer is delegated to make error-control. Downlink beamforming can serve a group of multicast users, guaranteeing a prescribe quality of service for each user. So, reliable data transmission is achieved, sending at the same time a single copy to all users of multicast group. According to this scenario, we assume the knowledge of the instantaneous channel state information (CSI). If this channel information passes to network layer and the users are scheduled during good channel condition with dynamic rate adaptation, substantial gain of network throughput can be achieved. In addition, radiation power of AP is minimized and thereby interference to neighboring cell is decreasing.

The drawback in such a multicast scenario is that receivers are found at different locations, near or away from AP. The constraint of minimum guarantee signal to noise ratio (SNR) for each receiver must be satisfied. In order to improve the performances of our system, multiple antennas at each receiver is proposed. Multiple antennas at the receiver permit beamsteering, which increases the SNR. Thus an increase in both data rate and range without any increase in transmit power is achieved. When MIMO are used, spatial multiplexing can be employed to allow additional data transmission. The design of beamformers for broadcast and several co-channel multicast groups has been previously proposed in [2] and [3]. However this approach is developed for a single carrier transmission over a frequency flat channel, without antenna arrays at user side. Our approach is referred to an OFDM system, compliant to IEEE802.11n. The frequency-selective fading channel is sliced into a number of narrowband channels, which are tagged to the respective subcarrier frequencies. A WLAN spans a small geographic area, typically a single building or a cluster. A transmission from one AP is receiving by all the users of the network. Thus, in a rich scattering environment, a single multicast group is implemented. Since 48 OFDM subcarriers spreading over 20 MHz bandwidth, we choice the most

attenuated. We minimize the total transmit power subject to providing at least a prescribed received SNR to each intended receiver for this subcarrier. If the above optimization problem is infeasible, users or subcarriers are dropped. Dynamic rate is employed to achieve higher throughput. It's adapted on the weakest subcarrier from all multicast group-users.

The rest of the paper is organized as follows: In the next section, we find the subcarriers for optimization. In section III algorithm of antenna arrays for mobile users is developed. Section IV formulates the beamforming optimization problem, while section V presents adaptation and scheduling techniques. In section VI, performances of proposed algorithms are extensively simulated. Section VII provides concluding remarks and areas for future investigation.

## II. OPTIMIZATION SUBCARRIERS

Multipath fading will be frequency selective. The received signal includes multiple versions of the transmitted waveform, which are attenuated and delayed in time. We model the radio channel as a linear filter,  $L$  taps with impulse response

$$h(\tau, t) = \sum_{l=0}^{L-1} \alpha_l(t) \delta_c(\tau - \tau_l) \quad (1)$$

where  $\alpha$  and  $\tau$  are attenuation and propagation delay. The filtering nature of the channel is caused by the summation of amplitudes and delays of the multiple arriving waves at any instant of time. The frequency response at time  $t$  is

$$H(f, t) = \sum_{l=0}^{L-1} \alpha_l(t) e^{-j2\pi f \tau_l(t)} \quad (2)$$

The contribution due to a particular path has linear phase in  $f$ . For multiple paths, there is a differential phase  $2\pi f \tau_l(t)$ . This implies that signals will be affected only at a part of the available frequency band. At certain frequencies it will be enhanced and will be completely partially suppressed at others frequencies. Frequency selective fading occurs when the delay spread  $T_d$  is larger than the symbol period  $T$ .  $T_d$  is defined as the difference in propagation time between the longest and shortest path. In other words, the last path arrives at  $T_d$  sec after the arrival of first path. A received symbol is influenced by previous symbol, which is termed as inter-symbol interference (ISI). When the bandwidth of the signal  $w = \frac{1}{T}$  is much larger than the coherence bandwidth  $w_c$  ( $w_c = \frac{1}{2T_d}$ ) the channel is frequency-selective. When  $w \ll w_c$  the channel is usually referred to as flat fading and a single filter tap is sufficient to present the channel.

Let us consider the receiving scheme where an AP equipped with  $M_r$  received antennas, uniformly spaced at distance  $d$ . If  $d_p$  is the path distance between the user and antenna array, wavefronts impinging on the array are planned, then impulse response of the channel which express baseband channel gain is

$$[h_1, \dots, h_{M_r}] = \alpha e^{\frac{-j2\pi f d_p}{c}} \left[ 1, e^{\frac{j2\pi d \sin \theta}{\lambda}}, \dots, e^{\frac{j2\pi d (M_r - 1) \sin \theta}{\lambda}} \right]$$

$\theta$  is the incident angle of plane wave and  $c$  is the speed of light. With MIMO-OFDM the ISI is removed by breaking the wideband channel into many narrowband subchannels. Each subchannel experiences flat fading and so can be treated as a flat fading MIMO channel [4]. Vector  $\left[ 1, e^{\frac{j2\pi d \sin \theta}{\lambda}}, \dots, e^{\frac{j2\pi d (M_r - 1) \sin \theta}{\lambda}} \right]$  depends on direction of arrival (DOA) of plane wave. DOA relies on scatters which are close to antenna arrays. The absolute delay  $2\pi f d_p$  does not affect our system. Antenna arrays form beams to detect or emit signals at directions of interest. Adaptive beamforming to different subchannels, only the gain of the antennas weights are changed according to attenuation factor  $\alpha$ . Par consequence, we minimize total transmit power subject to providing at least a prescribed SNR at each receiver, for the subchannel with minimum gain. Simultaneously this power guarantees the above constraints for all others subcarriers with greater gain.

Let's study a system with a single AP and  $K$  receivers. We indicate with  $I_{SC}$  the set of indexes corresponding to the subcarriers. Consider a MIMO system with  $M_t$  Tx antennas and  $M_r$  Rx antennas. Denoting  $H_k(n)$  an  $M_r \times M_t$  matrix whose entry  $(i, j)$  is the complex flat-fading coefficient at subcarrier  $n \in I_{SC}$  between the  $j$ th transmit antenna and  $i$ th receive antenna for user  $k$ . From basic properties of linear algebra, every matrix  $H_k(n)$  can be factorized by its singular value decomposition (SVD)

$$H_k(n) = U_k(n) D_k(n) V_k^H(n) \quad (3)$$

where  $D_k(n)$  is an  $M_r \times M_t$  matrix with only non zero elements given by  $D_k(n)[i, i] = \sqrt{\lambda_{ki}(n)}$ .  $i = 1, 2, \dots, q_k(n)$  with  $q_k(n) = \min(M_t, M_r)$  the rank of  $H_k(n)$  correspond to the number of spatial degrees of freedom. The SVD is an appropriate way of diagonalizing the matrix  $H_k(n)$ , which lead to a number of parallel channels (eigen modes). The power gain of  $i$ th mode is  $\lambda_{ki}(n)$ . The columns  $u_{ki}(n)$  and  $v_{ki}(n)$  of  $U_k(n)$  and  $V_k(n)$  are orthonormal so that

$$U_k^H(n) U_k(n) = I_{M_r} \quad V_k^H(n) V_k(n) = I_{M_t} \quad (4)$$

the columns  $u_{ki}(n)$  and  $v_{ki}(n)$  are the optimum weights at the Rx and Tx antennas for  $i$ th eigen mode [5]. Then, the received power at user  $k$  is

$$P_k(n) = |U_k^H(n) H_k(n) V_k(n)|^2$$

$$P_k(n) = \sum_{i=1}^{q_k(n)} D_k^2(n) [i, i] \quad (5)$$

The subcarrier  $n_{min}(k)$ , which corresponds to weakest gain at each user  $k$ , is taken from

$$n_{min}(k) = \arg \min_{n \in I_{SC}} \sum_{i=1}^{q_k(n)} D_k^2(n) [i, i] \quad (6)$$

$\forall k \in \{1, 2, \dots, K\}$  Therefore, each user is characterized from a subchannel  $\nu_k = n_{min}(k)$  with  $\nu_k \in \{1, 2, \dots, N\}$ .

Finally, we define the set

$$I_{\min} = \{\nu_1, \nu_2, \dots, \nu_k, \dots, \nu_K\}.$$

### III. RECEIVER ANTENNA ARRAYS

The separability of the MIMO channel relies on the presence of rich multipath which needs to make the channel spatially selective. Under these conditions, it's possible to transmit  $\min(M_t, M_r)$  independent data streams simultaneously over the eigenmodes of a matrix channel  $H$ . Clearly, the rank of  $H$  is always both less than the number of Tx antennas and less than the number of Rx antennas.

The columns of  $V_k$  are used as Tx weight vectors and the rows of  $U_k^H$  as Rx vectors. This strategy is referred as Eigenvector Steering (ES)[6]. A second strategy is the beamforming. Beamforming provides diversity and array gain via coherent combining of the multiple signal paths. The same symbol is weighted by  $v_{ki}$  and sent over each transmit antenna. At receiver, the signal is weighted by  $u_{ki}^*$ . We choose  $v_{ki}$  and  $u_{ki}^*$  which correspond to the maximum singular value of  $H_k$ . All the spatial multiplexing benefit of MIMO channel is sacrificed to maximize the reliability. In order to receive scattering energy from all reflected paths and consequently achieve higher reception diversity gain, we apply to our design the rows of  $U^H(\nu_k)$  as weights at each  $i$  element of the received array with  $i=1,2,\dots,M_r$ .

Reference training sequences are transmitted from AP in 4 pilot tones to estimate the channel matrix  $H_k(n)$ . The location of pilot carriers is :  $l=-21,-7,7,21$ . User  $k$  measures subchannel matrix  $H(\nu_k)$  and send it to AP through MIMO channel Measurement frame defined in 802.11n[7]. The attenuation of data subcarriers between these pilot symbols are typically estimated/interpolated using time correlation property of fading channel [8].

### IV. BEAMFORMING OPTIMIZATION FOR AP

The beauty of OFDM is the low implementation complexity. At the transmitter, the subcarriers are not individually filtering nor amplifying. Therefore, the beamformer must multiply by the same complex weights all subchannels. We treat the problem by transforming the frequency selective channel into a frequency flat channel describing propagation loss and phase shift by  $H(\nu_k)$  with  $\nu_k \in I_{\min}$ . We have assumed that channel impulse response changes at a rate much slower than the transmitted baseband signal. The received signal is given by

$$y(\nu_k) = H(\nu_k)x + \eta(\nu_k) \quad (7)$$

where  $x$  is the vector of signals emitted from AP and  $\eta(\nu_k)$  the noise present at user  $k$  for subchannel  $\nu_k$ . If Rx matrix is  $U(\nu_k)$  and  $v \in \mathbb{N}_t \times 1$  is the applying beamforming weight vector to  $M_t$  antenna elements then the transmit signal is given by  $x = vs$  where  $s$  denotes the information signal directed to  $K$  receivers. Assuming that  $s$  is zero mean, temporally white with unit variance, the transmit signal is equal to  $x = v$  and total radiated power is  $P = \|v\|_2^2$ . The estimated signal at receiver  $k$  is given by

$$\hat{s}(\nu_k) = U^H(\nu_k)y(\nu_k) \quad (8)$$

and the received power is

$$P(\nu_k) = |U^H(\nu_k)H(\nu_k)v|^2 \quad (9)$$

If the noise is zero mean with variance  $\sigma_k^2$  then noise power is  $\sigma_k^2 \|U(\nu_k)\|^2$ . Given  $U(\nu_k)$ , the pose problem is to generate an optimal downlink beamforming at AP, minimizing at the same time the total transmitted power and guarantying prescribed SNR constraint  $\gamma$  for all users.

$$\begin{aligned} \text{Q} \quad & \min \|v\|_2^2 \\ \text{s.t.} \quad & \frac{|U^H(\nu_k)H(\nu_k)v|^2}{\sigma_k^2 \|U(\nu_k)\|^2} \geq \gamma \\ & \|U(\nu_k)\|^2 = 1 \quad \forall k \in \{1, 2, \dots, K\} \quad \forall \nu_k \in I_{\min} \end{aligned}$$

This problem was found NP-hard for general channel vector [2]. Let introduce complex vector  $r(\nu_k) = U^H(\nu_k)H(\nu_k)$   $M_r \times M_t$ , define  $V = vv^H$ ,  $R(\nu_k) = r^H(\nu_k)r(\nu_k)$  and use that  $|r(\nu_k)|^2 = v^H r^H(\nu_k)r(\nu_k)v = \text{trace}\{VR(\nu_k)\}$ .

We apply the following constraints: a)  $\text{rank}V = 1$  due to the fact that  $V = vv^H$  b)  $V \geq 0$  and  $V = V^H$ , which mean that  $V$  is symmetric, positive, semidefinite matrix. In the general case, the constraint  $\text{rank}V = 1$  is not convex. Ignore the associated non convex constraint, the original problem is relaxed to a suitable Semi Definite Programming (SDP) problem. We introduce  $K$  real "slack" variables  $s_k$  and we underline the fact that the terms in denominator of linear inequalities are all non-negative; we take the relaxation problem R.

$$\begin{aligned} \text{R} \quad & \min \text{trace}(V) \\ \text{s.t.} \quad & \text{trace}(R(\nu_k)V) - s_k = \gamma \sigma_k^2 \\ & V_g \geq 0 \quad V_g = V_g^H \quad s_k \geq 0 \\ & \forall k \in \{1, 2, \dots, K\} \quad \forall \nu_k \in I_{\min} \end{aligned}$$

Problem R can be efficiently solved by any SDP solver, such as SeDuMi [9], based on interior point methods. The complexity per iteration is  $O((M_t^2 + K)^3)$  and for solution accuracy  $\epsilon$  SeDuMi gives  $O(\sqrt{M_t^2 + K} \log 1/\epsilon)$  worst-case iteration bound. The relaxed problem R provides only lower bounds on the optimal solution. Our solution  $V_{\text{opt}}$  is not rank-one in general. A randomization procedure (mentioned as rand C in [2]) is employed to generate candidate beamforming  $v$ . The computation cost is negligible. At the beginning SVD is used in  $V_{\text{opt}} = U\Sigma U^H$ . We put  $v = U\Sigma^{1/2}w_g$ , where  $w_g$  is a Gaussian variable with  $w_g \sim N(0, 1)$  to insure that  $E[vv^H] = V_{\text{opt}}$ . From computing feasible points, the vector with minimum  $\|v\|_2^2$  is chosen.

### V. RATE ADAPTATION AND SCHEDULING

An interesting feature of 802.11 WLAN is adaptive modulation and coding (AMC), also known as link adaptation

TABLE I. MODE –DEPENDENT PARAMETERS

Mode	Modulation	Coding Rate (R)	Coded bits per subcarrier	Minimum Sensitivity (dBm)
1	BPSK	1/2	1	-80
2	QPSK	1/2	2	-77
3	QPSK	3/4	2	-75
4	16-QAM	1/2	4	-72
5	16-QAM	3/4	4	-68
6	64-QAM	2/3	6	-64
7	64-QAM	3/4	6	-63
8	64-QAM	5/6	6	-62

By selecting a constellation from BPSK to 64-QAM and a convolutional coding rate for each subchannel, the PHY data rate is adapted to the link quality. The link adaptation is based on the knowledge of SNR, which is assumed to be perfectly known. At Table I, minimum receiver sensitivity is the power at the antenna port of receiver for which packet error rate (PER) is less than 1% for PHY layer service data unit(PSDU) or payload length equal to 4095 bytes. For bandwidth  $W=20$  MHz, the noise power at the input of each receiver is  $N=N_0W$  with  $N_0=KT$ .  $K=1.38 \cdot 10^{-23}$  J/K is the Boltzmann's constant and  $T=290^\circ$  K is the absolute room temperature.  $N(\text{dBm})=-174+73=-101$ . If receiver noise figure is  $NF=6\text{dB}$ , total noise floor is  $N_{\text{floor}}(\text{dBm})=-101+6=-95$ . Minimum sensitivity for each mode requires minimum SNR equal to  $\text{SNR}(\text{dB})=\text{Sensitivity}(\text{dBm})-N_{\text{floor}}(\text{dBm})$ . Par consequence, in our simulation, the received SNR constraint is set to  $\gamma=15\text{dB}$  and the additive noise power at  $\sigma_k^2=-95$  dBm.

Total OFDM symbol duration is  $T_{\text{tot}}=T_g+T_u=16+64=80$  samples.  $T_g$  is the guard interval duration and  $T_u$  the useful symbol duration. Since channel duration is 20MHz, sampling period is  $T_s=1/20 \mu\text{s}$  and  $T_{\text{tot}}=4 \mu\text{s}$ . When guard interval  $T_g=16 \cdot 0.05=0.8 \mu\text{s}$  is longer than the excess delay of radio channel, ISI is eliminated. The users have different link rates because they have different fluctuations in channel quality. The multicast rate is adapt to subchannel with the lowest link rate due to the shared channel characteristic. The minimum data rate for each user is given for mode 1.  $R_{\text{min}}=48\text{subcarriers} \times 1/2\text{bit} / 1\text{coded bit} \times 1\text{coded bit} / \text{subcarrier symbol} \times 1\text{subcarrier} / 4 \cdot 10^{-6} \text{sec} = 6 \text{ Mbits/s}$ . The maximum data rate is achieved for mode 8.  $R_{\text{max}}=48\text{subcarriers} \times 5/6 \text{ bit} / 1\text{coded bit} \times 6\text{coded bit} / \text{subcarrier symbol} \times 1\text{subcarrier} / 4 \cdot 10^{-6} \text{sec} = 60\text{Mbits/s}$ .

In the scheduling process, user  $k$  allocated to multicast group if satisfy the below inequality:

$$\text{SNR} = \frac{|U^H(\nu_k)H(\nu_k)v|^2}{\sigma_k^2} \geq \gamma \quad (10)$$

Beamforming optimization problem  $Q$  can easily become infeasible. In this situation, scheduler calculates using (6) the minimum  $n_{\text{min}}(k)$  for all  $k \in \{1, 2, \dots, K\}$  and

- Drop the user  $k$  (admission control) if scheduler policy is the maximum throughput.
- Remove subchannel  $n_{\text{min}}$  if scheduler policy is the maximum number of users that can be served (fairness).

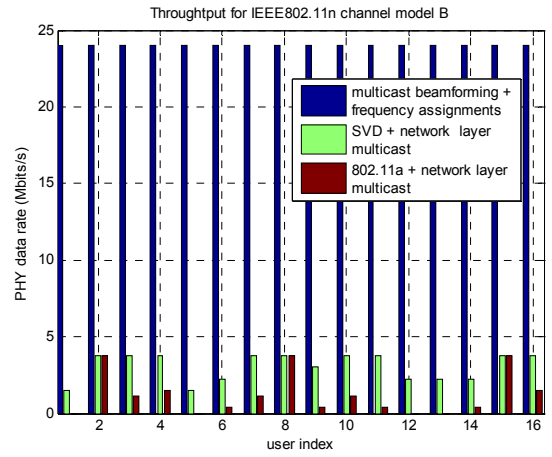


Figure 1. Throughput for different users

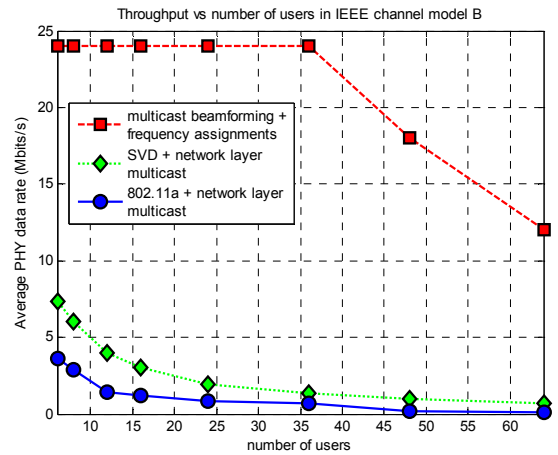


Figure 2. Throughput versus number of users

## VI. PERFORMANCE EVALUATION

In this section, we present some performance results for a realistic WLAN. We simulate in MATLAB environment an AP consists of  $M_t=4$  antenna elements and up to 64 users of  $M_r=4$ . IEEE802.11n channel model B in NLOS conditions is used to calculate  $h$  parameters [10]. Implementation is available from L. Schumacher [11]. Antenna geometries, distance between Tx and Rx, carrier frequency, correlation coefficient type(complex/real), Doppler spread can be set. Antenna arrays are uniform linear (ULA) with distance between elements  $d=\lambda/2$ . Cell radius is  $R=100\text{m}$  and operation frequency 5.2 GHz for path loss calculation. Due to approximation of numerical results, we realize 100 Monte Carlo runs for different locations of users. At each location, 107 interpolated samples are collected for a time varying channel due to the Doppler effect. If we take the speed of moving scattering environment equal to  $v_0=1.2 \text{ Km/h}$  maximum Doppler shift is  $f_m=v_0 f/c = 208 \text{ Hz}$ . Coherence time due to Doppler spread is  $T_c \approx 1/f_m = 40808 \mu\text{s}$ , that is, 1000 times slower than the transmitted OFDM symbol. The channel may be assumed to be static. Finally, at each run 1000 Gaussian randomization samples to solve rand C problem are generated. Received power in (9) is normalized because information signal  $s$  is zero mean with unit variance ( $\|s\|^2=1$ ).

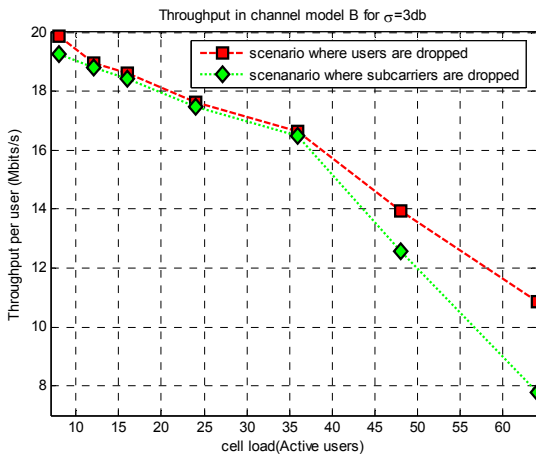


Figure 3. Throughput per user vs cell load

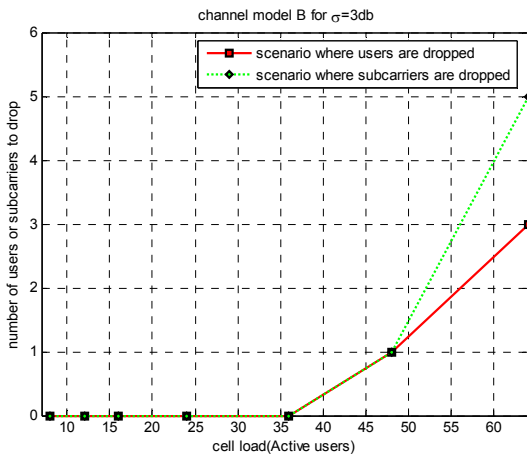


Figure 4. Number of removed users or subcarriers

Accordingly, received power must be multiplied by  $P_t$ , where  $P_t$  is the transmit power of AP equal to 500mW. Fig 1 shows the average physical layer data rate per user for different users while fig. 2 presents average PHY data rate versus number of users. Our propose scheme for beamforming and frequency assignment optimization is compare with a WLAN 802.11n and 802.11a both without cross-layer information. SVD technique described in section II is proposed for IEEE 802.11n standard [6,12]. In order to obtain maximum throughput, we adjust the prescribe SNR  $\gamma$  to simulation in order to achieve average transmit power 500 mW for all samples. The peak transmitted power is taken equal to 2W. For reason of simplicity ,simulation results in fig. 1 and fig. 2 are taken assuming that channel is flat for entire band. We remark that the first technique achieves a noticeable improvement. Fig 3 and fig. 4 compare the two scheduling techniques proposed in section V. In these case studies,  $SNR_k(n)$  for user k and subcarrier n follows Gaussian distribution with mean value calculated from simulation for flat frequency channel and standard deviation  $\sigma=3\text{dB}$  as in [13]. The two techniques are equivalent up to  $K=36$  users.

## VII. CONCLUSION

In this paper, we have investigated the efficiency of multicast beamforming optimization over IEEE802.11n WLAN. In simulation, we achieve a throughput per user up to 24 Mbits/s. Two scheduling schemes have been joined. The protocol a) rejects users that have bad channel conditions, when our design criterion is to maximize throughput b) removes subcarriers when our design criterion is the satisfaction of users.

A possible direction of future work would be to investigate physical layer multicasting not only to IEEE802.11n but also to various wireless communications, including ad-hoc networks and fourth generations mobile communications based on OFDM scheme with hundreds of subcarriers.

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